Inhomogeneous Cosmology, Inflation and Late-Time Accelerating Universe

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Abstract

An inhomogeneous cosmology describing a spacetime without symmetry is shown to be able to inflate the early universe and explain the late-time acceleration of the universe without a cosmological constant and negative pressure dark energy and avoid the coincidence problem.

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1 Introduction

The problem of explaining the acceleration of the universe as determined by supernovae data and the cosmic microwave background (CMB) data is one of the most significant outstanding problems in modern physics and cosmology [1, 2, 3, 4]. The standard explanation is either based on postulating a cosmological constant Λ or assuming that some form of uniform dark energy with negative pressure exists in the universe [5]. The postulate of a cosmological constant or a form of quintessence leads to the severe fine-tuning associated with the "coincidence problem": Why is the universe undergoing a period of acceleration now? The explanation as to why the cosmological constant is zero or very small has led to a crisis in physics and cosmology. One possible solution of the cosmological constant problem is that there exists a symmetry between positive and negative energy states in quantum field theory. A generalized charge conjugation invariance of the vacuum and the postulate that negative energy bosons satisfy a para-statistics Pauli exclusion principle can solve the cosmological constant problem [6].

Inflationary models postulate a false vacuum which decays slowly to the true vacuum inducing a large enough inflationary period to solve the horizon and flatness problems [7]. A standard scenario involves an inflaton potential with a slow roll of an inflaton particle to the true vacuum state. The inflation models require significant fine-tuning and suffer from a lack of a convincing particle physics interpretation of the inflaton particle.

In the following, we shall investigate an inhomogeneous cosmological solution of Einstein's field equations obtained by Szekeres [8], for an irrotational dust dominated universe and subsequently generalized by Szafron [9] and Szafron and Wainright [10, 11] to the case when the pressure p is non–zero. This cosmological model contains the Friedmann-Lemaître-Robertson-Walker (FLRW) cosmology and the spherically symmetric Lemaître-Tolman-Bondi (LTB) model [12, 13, 14] as special solutions. Recently, the LTB model was used to obtain an explanation for the non-Gaussian behavior observed in the WMAP data [3, 4] and give a possible explanation for the late-time acceleration of the universe [15, 16, 17]. A class of spherically symmetric Stephani models [11] has been considered as inhomogeneous cosmological models that can lead to accelerated expansion [18, 19].

A criticism of the LTB and Stephani models is that they assume a spherically symmetric universe with one spatial degree of inhomogeneity, requiring a center of the universe or a restricted description of the late-time, non-linear regime with voids and collapsing matter. The Szafron model describes a more general inhomogeneous spacetime with no prescribed symmetry [20].

We will show that the Szafron inhomogeneous cosmology in the early universe near the Planck time can lead to an accelerating inflationary period without a large initial vacuum energy and with $\Lambda = 0$. Moreover, at late times in the non-linear regime with galaxy structure and voids the cosmology can explain the acceleration of the universe without negative pressure dark energy or a cosmological constant and avoid the coincidence problem.

2 The Inhomogeneous Cosmological Solution

The metric takes the form[1]

$$ds^2 = dt^2 - \exp(2\alpha)dz^2 - \exp(2\beta)(dx^2 + dy^2), \tag{1}$$

where $\alpha = \alpha(x, y, z, t)$ and $\beta = \beta(x, y, z, t)$ are to be determined from Einstein's field equations

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}, \tag{2}$$

where $T_{\mu\nu}$ denotes the perfect fluid energy-momentum tensor

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu}, \tag{3}$$

and Λ is the cosmological constant. Moreover, ρ denotes the energy-density of matter, p the pressure and u_μ the velocity field of the fluid which is normalized to

[1] We use units with the speed of light c=1.

$u^\mu u_\mu = 1$. The coordinates are assumed to be comoving so that $u^\mu = \delta_0^\mu$ and $\dot{u}^\mu = 0$ where $\dot{u}^\mu = du^\mu/dt$. Szafron [9, 11] solved the Einstein equations with $p = p(t) \neq 0$ and $\Lambda = 0$, generalizing the dust solution of Szekeres [8]. There are two classes of solution $\beta' \neq 0$ and $\beta' = 0$ where $\beta' = \partial\beta/\partial z$. The more general solution $\beta' \neq 0$ is given by

$$\exp(\beta) = R(z,t)\exp(\nu(x,y,z)), \tag{4}$$

$$\exp(\alpha) = h(z)\exp(-\nu(x,y,z))\partial\exp(\beta)/\partial z, \tag{5}$$

where

$$\exp(-\nu(x,y,z)) = A(z)(x^2+y^2) + 2B_1(z)x + 2B_2(z)y + C(z). \tag{6}$$

The $A(z), B_1(z), B_2(z), C(z)$ and $h(z)$ are arbitrary functions of z. Moreover, R satisfies the equation

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = -8\pi G p, \tag{7}$$

which has the same form as one of the Friedmann equations in FLRW cosmology except that $R = R(z,t)$ and $k = k(z)$. The function $k(z)$ is determined by

$$A(z)C(z) - B_1^2(z) - B_2^2(z) = \frac{1}{4}\Big[\frac{1}{h^2(z)} + k(z)\Big]. \tag{8}$$

The case $\beta' \to 0$ is singular. Eq.(7) can be formally integrated once $p(t)$ is specified.

We shall consider in the following, the simpler Szafron solution with $\beta' = 0$ given by

$$\exp(\beta) = \frac{R(t)}{1 + \frac{1}{4}k(x^2+y^2)}, \tag{9}$$

$$\exp(\alpha) = \lambda(z,t) + R(t)\Sigma(x,y,z), \tag{10}$$

$$\Sigma = \frac{\frac{1}{2}U(z)(x^2+y^2) + V_1(z)x + V_2(z)y + 2W(z)}{1 + \frac{1}{4}k(x^2+y^2)}. \tag{11}$$

Now k is a constant, $U(z), V_1(z), V_2(z)$ and $W(z)$ are arbitrary functions of z and $R(t)$ is determined by the Friedmann equation

$$2\frac{\ddot{R}(t)}{R(t)} + \frac{\dot{R}^2(t)}{R^2(t)} + \frac{k}{R^2(t)} = -8\pi G p(t). \tag{12}$$

We can choose without loss of generality $W(z) = 0$ and $\lambda(z,t)$ is determined by

$$\ddot{\lambda}R + \dot{\lambda}\dot{R} + \lambda\ddot{R} = -8\pi G\lambda R p + U(z). \tag{13}$$

The matter density equation is given by

$$2\Big[\lambda(z,t)\frac{\ddot{R}(t)}{R(t)} - \ddot{\lambda}(z,t)\Big]\exp(-\alpha) + 3\frac{\dot{R}^2(t)}{R^2(t)} + \frac{3k}{R^2(t)} = 8\pi G\rho. \tag{14}$$

Eqs.(12) and (14) can be solved once the pressure $p = p(t)$ is specified. The FLRW spacetimes are obtained when $\lambda(z,t) = U(z) = 0$. When $U(z) = 0$ and $V_1(z) = V_2(z) = 0$ the model possesses a 3-dimensional symmetry group acting on 2-dimensional orbits. The symmetry is spherical, plane or hyperbolic when $k > 0, k = 0$ or $k < 0$, respectively. There does exist, in general, a quasi-symmetry for the surfaces $t =$ const., $z =$ const. which have constant curvature with non-parallel planes.

3 The Inflationary Solution

Let us now consider the very early universe and choose for simplicity the spatially flat solution $k = 0$. From (14) we obtain

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G\rho}{3} + \frac{2}{3}\Big(\ddot{\lambda} - \lambda\frac{\ddot{R}}{R}\Big)F, \tag{15}$$

where

$$F \equiv \exp(-\alpha) = \frac{1}{\lambda + R\Sigma}. \tag{16}$$

Let us now assume that

$$X \equiv \frac{2}{3}\Big(\ddot{\lambda} - \lambda\frac{\ddot{R}}{R}\Big)F > 0, \tag{17}$$

that $X \sim$ const. and $X > 8\pi G\rho/3$. Then, we can have a de Sitter inflationary period with

$$R(t) \propto \exp(Ht), \tag{18}$$

where

$$H \equiv \dot{R}/R = \sqrt{X}. \tag{19}$$

In this scenario, the inflation of the early universe is caused by the *intrinsic inhomogeneity of spacetime* near the Planck time, $t \sim t_{Pl}$, determined by the solution of the metric (1) using Einstein's field equations and $\Lambda = 0$. Thus, we are able to obtain an inflationary period of the early universe from Einstein's field equations without postulating extra degrees of freedom such as inflaton fields and a fine-tuned inflaton potential.

4 Acceleration of the Late-Time Matter Dominated Universe

The inflationary period initiated by the early inhomogeneous universe inflates away the inhomogeneity and anisotropy and produces the seeds at the surface of last scattering with $\delta T/T \sim \delta\rho/\rho \sim 10^{-5}$ where T denotes the temperature. When galaxies, clusters of galaxies and voids have formed in the late-time, non-linear regime for a redshift $z < 5$, we now have a new inhomogeneous matter dominated spacetime which will be described by our exact inhomogeneous cosmology.

Setting the pressure $p = 0$ and assuming a spatially flat universe, $k = 0$, we get from (12)and (14):

$$H^2 = \frac{8\pi G\rho}{3} + \frac{2}{3}\left(\ddot{\lambda} - \lambda\frac{\ddot{R}}{R}\right)F. \tag{20}$$

and

$$\frac{\ddot{R}}{R} = -\frac{4\pi G\rho + \ddot{\lambda}F}{3(1 - \frac{1}{3}\lambda F)}. \tag{21}$$

The universe will undergo an acceleration with $\ddot{R} > 0$ when $\lambda F > 3$ and

$$4\pi G\rho + \ddot{\lambda}F > 0. \tag{22}$$

From (20) we have

$$\Omega_M + \Omega_{\rm INH} = 1, \tag{23}$$

where

$$\Omega_M = \frac{8\pi G\rho}{3H^2}, \quad \Omega_{\rm INH} = \frac{2\left(\ddot{\lambda} - \lambda\frac{\ddot{R}}{R}\right)F}{3H^2}. \tag{24}$$

Let us expand $R(t)$ in a Taylor series

$$R(t) = R[t_0 - (t_0 - t)] = R(t_0)\left[1 - (t_0 - t)\frac{\dot{R}(t_0)}{R(t_0)} + \frac{1}{2}(t_0 - t)^2\frac{\ddot{R}(t_0)}{R(t_0)} - ...\right]$$

$$= R(t_0)\left[1 - (t_0 - t)H_0 - \frac{1}{2}(t_0 - t)^2 q(t_0)H_0^2 - ...\right], \tag{25}$$

where t_0 denotes the present epoch and $H_0 = \dot{R}(t_0)/R(t_0)$. We have

$$q = -\frac{\ddot{R}R}{\dot{R}^2}. \tag{26}$$

By substituting for $\ddot{R}$ from Eq.(21), we obtain

$$q = \frac{4\pi G\rho + \ddot{\lambda}F}{3H^2(1 - \frac{1}{3}\lambda F)}. \tag{27}$$

The deceleration parameter q can be negative when $\lambda F > 3$ signaling a late-time accelerating universe.

A spatial averaging $\langle q\rangle$ of the deceleration parameter can be performed that smooths out the inhomogeneity and avoids the no-go theorem obtained from the local fluid form of the Raychoudhuri equation [16, 17].

5 Conclusions

From the inhomogeneous geometry of spacetime, we have obtained the initial inflation of the universe and the late-time acceleration of the expansion of the universe from an inhomogeneous solution of Einstein's field equations. This avoids the postulate of a cosmological constant and a false vacuum energy to initiate the inflationary period and the late-time acceleration. There exist two phases of acceleration in the history of the universe: 1) the exponential expansion described by a de Sitter solution caused by the inhomogeneous very early universe near the Planck time, 2) the acceleration of the universe during the non-linear, inhomogeneous late-time period when galaxies, clusters of galaxies and voids have formed. In between these two phases of evolution, the universe is described by the homogeneous and isotropic FLRW cosmology with small anisotropic fluctuations in the CMB at the surface of last scattering. Because the late-time acceleration is caused by the inhomogeneous structure of galaxies, clusters and voids there is no "coincidence problem".

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